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Water–Energy–Food nexus evaluation using an inverse approach of the graph model for conflict resolution based on incomplete fuzzy preferences

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ABSTRACT

With the continuous development of economic globalization and the increasing strengthening of human exchanges, the Water–Energy–Food (WEF) nexus evaluation has become a new research area. The purpose of this paper is to develop an inverse approach of graph model for conflict resolution (GMCR) to the conflict problems of WEF nexus evaluation in real life. Specifically, due to lack of information, some decision makers' (DMs') preferences over states may be incomplete fuzzy preference relations. Therefore, an algorithm is devised to amend the incomplete fuzzy preference relation to the complete fuzzy preference relation. Subsequently, a complete ordinal score vector is proposed to describe the preference ranking over different states based on the complete fuzzy preference relation. Moreover, in the framework of the inverse approach of GMCR, some mathematical models with the least constraint conditions are proposed to obtain all the required preference relations for opponent DM, which are required to make a given state be stable under four basic stability definitions. Finally, WEF nexus evaluation in Shandong province is illustrated to demonstrate the usefulness of the inverse approach of GMCR with the incomplete fuzzy preference relations.

1. Introduction

In the 21st century, with the rapid growth of population and excessive consumption of resources, the concept of sustainable development has become a global consensus issue. Recently, more and more scholars have begun to study the complex relationships between water, energy, and food (WEF) [1,2]. In November 2011, to explore the development of green economy, the federal government of Germany held an international conference to explore the security of WEF nexus in Bonn. The conference focused on the necessity of comprehensive solutions, which were used to ensure the supply of resources and achieve rapid development of the economy. In general, the research field of the WEF nexus evaluation mainly focuses on two aspects: one is to reveal the internal relationship of the WEF nexus; the other one is to focus on the description and strategy of the dynamic change of WEF nexus evaluation in the context of ecological environment and social change.

To the best of the authors' knowledge, a typical problem of WEF nexus evaluation is how to design a reasonable water resource allocation project, which can allocate the limited water resources to the energy industry and food industry for maximizing the benefits. Thus, in coordinating the distribution of water, energy, and food resources among different government departments, DMs from different departments and fields play an incomparable role. In general, due to the difference in responsibilities and cultural backgrounds of each DM, the WEF nexus evaluation among different DMs in a certain situation can be considered a conflict problem. This also means that solving the WEF nexus evaluation conflict problem is an important step to obtain the optimal resource allocation scheme recognized by all DMs. In other words, in real-life WEF nexus evaluation conflict problems, different DMs are invited to deal with the conflict problem, evaluate and analyze the limited set of possible solutions, and then calculate the optimal solution by using the existing conflict analysis methods [3,4]. In general, the research of WEF nexus evaluation mainly concentrates on the area of engineering and system. Thus, it is more interesting and meaningful to study the WEF nexus evaluation from the perspective of management.

Conflict is a universal phenomenon in political, economic, military, cultural, and other fields. To better analyze and solve

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the conflict, many effective analysis models and methods have been introduced. Compared with the existing game-theoretical methods, such as game theory [5], metagame analysis [6], and conflict analysis [7], a simple, flexible, and comprehensive approach, called the graph model for conflict resolution (GMCR), which was proposed by Kilgour, et al. [8] and Fang, et al. [9] to model and analyze the conflict problems. Nowadays, GMCR is widely used in solving the real-world conflict by researchers and practitioners [10–13]. According to the specific solution step of GMCR, it can be classified into three stages: input, analysis, and output. Among them, the analysis stage is considered the most important step. Thus, many stability definitions have been proposed to identify whether a state is stable or not in the analysis stage. There are four basic stability definitions called Nash stability (Nash) [14,15], general metarationality (GMR) [6], symmetric metarationality (SMR) [6], and sequential stability (SEQ) [7].

In general, based on the basic stability definitions, the research of GMCR is mainly to determine whether a given state is stable for each DM. This is called the forward perspective problem of the graph model [16–18]. On the contrary, in some conflict negotiations, the analyst may wish to determine all the possible preference relations for each DM, which can be used to obtain the desired resolution, and it is called the inverse perspective problem of GMCR. It is a challenge to obtain the preference ranking over states by using inverse analysis. At present, there are fewer achievements in the research of the inverse approach of GMCR. Sakakibara, et al. [19] firstly emphasized the importance of the inverse problem of the graph model by using the logical representation. Kinsara, et al. [20] used the enumeration method to study the inverse problem of graph model. According to the existing optimization procedure, Wu, et al. [21] introduced a mediation tool to determine minimal priority adjustment, which is used to obtain the required preference relations. Wang, et al. [22] studied the matrix representations of the inverse problem of graph model. An integer programming approach is proposed to solve the inverse perspective problem of graph model [23]. The inverse analysis of graph model is solved by using genetic algorithm [24]. To achieve desired equilibrium state, the required preference relation for DM is determined by using a cost optimization model [25]. Up to now, there is no research on the WEF nexus evaluation problems by using the inverse approach of GMCR. This work is to fill this gap.

In the input stage of GMCR, the preference relations over states are always considered crisp and complete. However, due to the inherent uncertainty of human cognition, the preference relations of some DMs in conflict problems may be partially known. That means, in real-life conflict, the existing information of focus DM may be an uncertain preference relation, in which some elements are unknown or unclear. Wu, et al. [26] first integrated the incomplete fuzzy preference relation (IFPR) into the graph model. Li, et al. [27] and Li, et al. [28] proposed the graph model with unknown preferences to resolve the conflict problems at the same time. For more research on the graph model with uncertain preferences, please refer [29–32]. Additionally, many studies have focused on the IFPR [33–37]. Thus, how to solve the conflict problem with IFPR is meaningful work.

In this paper, the IFPR is introduced to describe the uncertain preference of DM in conflict problems, that is to say, some of the elements in the fuzzy preference relation (FPR) are missing. An algorithm is proposed to supplement an IFPR into a complete FPR. Then, a complete ordinal score vector can be obtained based on the complete FPR. As a result, a set of mathematical models with least constraint conditions has been introduced to calculate the possible preference relations for opponent DM, which are required to make a given state be stable under the basic stability definitions. The main contributions of this paper are:

(1) The paper studies the WEF nexus evaluation conflict problem with IFPR, which is more practical than the traditional model.

(2) Compared with the existing inverse approach of the graph model, the proposed method needs less known preference relations, which provides a new perspective to enrich and develop the theoretical framework of GMCR.

(3) The paper studies WEF nexus evaluation problems from the perspective of management, and analyzes the behavior of DMs in conflict problems.

The remainder of this paper is organized as follows. In Section 2, the basic structure of GMCR is introduced. In Section 3, the IFPR is introduced and an algorithm is designed to amend the missing elements in an IFPR. Then, the mathematical model of inverse GMCR with IFPR is introduced in Section 4. In Section 5, a case study of the WEF nexus evaluation conflict in Shandong province is illustrated to show how to solve the real-life conflict problem with IFPR, the implications of the study are shown from the perspective of management, and the limitations of research work are described. Section 6 shows the computational complexities of different inverse approaches of GMCR. Finally, some conclusions and directions for future work are introduced in Section 7.

2. Preliminaries

In this section, some basic definitions and terminologies of graph model are introduced.

2.1. The basic structure of GMCR

Let $N = \{1, 2, \ldots, n\}$ be a set of DMs and $S = \{s_1, s_2, \ldots, s_m\}$ be a set of feasible states. A graph model can be represented by $\{(G_k, \succsim_k), k \in N\}$, where $G_k = (S, A_k)$ represents the directed graph (a set of feasible states S is denoted as the vertices and A_k represents a set of directed arcs) for DM k and $\succsim_k$ denotes the preference relations over S for DM k. The direction of the arc indicates the possible movement in one step among the feasible states. Note that the moves may or may not be reversible. The binary relation $\{\succ_k, \sim_k\}$ represents the preference relation over S for DM k, which $s_i \succ_k s_j$ represents that DM k prefers s_i than s_j and the relation $s_i \sim_k s_j$ represents that DM k has the same preference between states s_i and s_j. In general, the preference relations of each DM are supposed to be crisp and complete.

Next, according to the preference degree over all feasible states for DM k, the ordinal score vector is proposed to represent the preference ranking over all feasible states for each DM.

Definition 1. The ordinal score vector of DM k is represented by a vector $V_k = (v_1, v_2, \ldots, v_m)$, where v_i represents the ordinal score value of the state s_i and is equal to an integer in the closed interval $[1, m]$, denoted as $V_k(s_i) = v_i$. The characteristic of ordinal score vector can be summarized as follows:

(1) The larger the score value of v_i, the higher the preference of the corresponding state s_i. Let s_i, s_j be two different states, DM k prefers s_i to s_j if and only if $v_i > v_j$.

(2) If two different ordinal score values are equal, it indicates that the corresponding two different states have the same preference degree. Thus, DM k has equal preference between s_i and s_j if and only if $v_i = v_j$.

(3) When each element of the ordinal score vector is different from each other, the vector is a strictly ordinal score vector.

For example, if $V_k = (3, 1, 2, 5, 4)$ represents the ordinal score vector over five states for DM k. $V_k(s_4) = 5$ represents the ordinal score value of state s_4 is 5 for DM k, which also means that the state s_4 is the fifth place in the ranking of five states from inferior to superior. Therefore, the preference ranking over all states for DM k would be $s_2 \prec s_3 \prec s_1 \prec s_5 \prec s_4$. In the following paper, the ranking of state in the ordinal score vector is determined according to the ranking of the same state of the weight vector, which is calculated from the complete pairwise comparison matrix for each DM. The rule is that the greater the value of each element in the weight vector, the greater the value of the corresponding state in the ordinal score vector.

In real-life conflict, the preference information of DMs may be partial. Thus, according to the partially known preference information of DM k, the incomplete ordinal score vector of the focus DM is described as follows.

Definition 2. The incomplete ordinal score vector of DM k is represented by the vector $V_k' = (v_1, v_2, *, \ldots, v_m)$, where v_i represents the ordinal score value of the state i and the "$*$" represents the unknown ordinal score value of a given state.

For simplicity, let $(i,j) \in A_k$ represents $(s_i, s_j) \in A_k$. In addition, let e_i be m-dimensional column vector, in which ith element of vector is 1 and other elements of vector are 0. Let e_i^T be the transposition of the vector e_i and m represents the number of states. The definitions of reachable list and improvement reachable list are shown as follows.

Definition 3 ([38]). Let $k \in N$ and $s \in S$. The reachable matrix is an $m \times m$ adjacency matrix, where

$$J_k(i,j) = \begin{cases} 1 & (i,j) \in A_k \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The $e_i^T \cdot J_k$ represents the ith row of the matrix J_k, and the $R_k(s_i)$ represents the reachable list $R_k(s_i) = \{s_j : J_k(i,j) = 1\}$.

Definition 4 ([38]). Let $k \in N$ and $s \in S$. The improvement reachable matrix is an $m \times m$ adjacency matrix, where

$$J_k^+(i,j) = \begin{cases} 1 & (i,j) \in A_k \text{ and } V_k(s_j) > V_k(s_i) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The $e_i^T \cdot J_k^+$ represents the ith row of the matrix J_k^+, and the $R_k^+(s_i)$ represents the improvement reachable list $R_k^+(s_i) = \{s_j : J_k^+(i,j) = 1\}$.

Remark 1. Given $|R_k(s)| = m$, there are 2^m cases according to whether the reachable state of the group $R_k(s)$ is improvement reachable.

It is evident that different DMs have different behavioral choices in responding to the conflict. Specifically, each DM has a different degree of foresight for making a decision in conflict and each DM may have a different degree of attitude for the risk of moving. Therefore, we fully consider different DM's attitudes of risk and combine moves and countermoves between states. To describe different DMs behaviors and their decision techniques, according to Definitions 3 and 4, four basic stability definitions are introduced within the framework of GMCR.

Definition 5 ([14,15]). Nash: Let $k \in N$ and $s \in S$, the state s is called Nash equilibrium for DM k iff $R_k^+(s) = \varnothing$, denoted as $s \in S_k^{Nash}$.

Definition 6 ([6]). GMR: Let $k \in N$ and $s \in S$, the state s is called GMR for DM k iff for each $s_1 \in R_k^+(s)$ there exists at least one $s_2 \in R_{N-\{k\}}(s_1)$ such that $s_2 \precsim_k s$, where $N - \{k\}$ represents all DMs excluding DM k, denoted as $s \in S_k^{GMR}$.

Definition 7 ([6]). SEQ: Let $k \in N$ and $s \in S$, the state s is called SEQ for DM k iff for each $s_1 \in R_k^+(s)$ there exists at least one $s_2 \in R_{N-\{k\}}^+(s_1)$ such that $s_2 \precsim_k s$, denoted as $s \in S_k^{SEQ}$.

Definition 8 ([7]). SMR: Let $k \in N$ and $s \in S$, the state s is called SMR for DM k iff for each $s_1 \in R_k^+(s)$ there exists at least one $s_2 \in R_{N-\{k\}}(s_1)$ such that $s_2 \precsim_k s$ and $s_3 \precsim_k s$ for all $s_3 \in R_k(s_2)$, denoted as $s \in S_k^{SMR}$.

Remark 2. When the conflict involves two DMs, a set of DMs N in Definitions 5–8 is simplified to $N = \{k, l\}$. $R_{N-\{k\}}$ and $R_{N-\{k\}}^+$ are rewritten as R_l and R_l^+, respectively.

3. IFPR and its completeness

In this section, the IFPR is introduced to describe the uncertain preference over all states for DM in the conflict problems. Then, an algorithm is designed to supplement IFPR.

3.1. IFPR

The fuzzy preference is an important branch of the fuzzy relation theory. Fuzzy preference as an analysis tool has been widely used in many fields, especially in engineering and management. As we all known, an FPR over all states is represented by a preference matrix and the elements of matrix indicate the degree of preference of one state over another state. The classical definition of FPR is shown as follows.

Definition 9 ([39,40]). Let $R = (r_{ij})_{m\times m}$ be an FPR, r_{ij} represents the preference level between state s_i and s_j, satisfying

$$r_{ij} + r_{ji} = 1, r_{ii} = 0.5, 0 \le r_{ij} \le 1, \quad \text{for all } i,j = 1,2,\ldots,m \tag{3}$$

Particularly,

(1) $r_{ij} > 0.5$ represents that the state s_i is preferred to s_j. $r_{ij} = 1$ represents that the state s_i is definitely preferred to s_j.

(2) $r_{ij} < 0.5$ represents that the state s_j is preferred to s_i. $r_{ij} = 0$ represents that the state s_j is definitely preferred to s_i.

(3) $r_{ij} = 0.5$ represents that the indifference between state s_i and state s_j.

In real life, due to the lack of information, experience, and knowledge, the preference relation of DM may be an IFPR. In other words, the preference values of focus DM are partially known, which constitutes an IFPR.

Definition 10 ([33]). An FPR matrix $R = (r_{ij})_{m\times m}$ is called the IFPR matrix if some of its elements cannot be given by the DM. The symbol "z_{ij}" is used to represent the unknown elements. All other elements of the matrix satisfy the condition (3).

As mentioned above, the FPR for each DM is a pairwise relation that describes the preference degree between one state and another state. The maximum preferred value of 1 represents that one state is absolutely preferred to another state. Especially, when the preferred value is greater than 0 and less than 1, it indicates that there must be some reasons to convince DM that one state of pair of states may be preferable to the other state. More importantly, according to $r_{ij} + r_{ji} = 1$, the value of $r_{ji} = 1 - r_{ij}$ can be interpreted as the degree to which the state s_i is not preferred over state s_j. Thus, a formal definition of the fuzzy relative strength of preference is proposed to describe the degree of preference of DM for a given state.

Definition 11 ([17]). Let $k \in N$, $i,j = 1,2,\ldots,m$, and r_{ij}^k be the preference degree of state s_i over state s_j for DM k. Then, the fuzzy relative strength of preference between state s_i and state s_j for DM k is represented as $a^k(s_i, s_j) = r_{ij}^k - r_{ji}^k$.

Obviously, the value range of $a^k(s_i, s_j)$ is from -1 to 1. Specially, $a^k(s_i, s_j) = -1$ indicates that state s_j is absolutely preferred to state s_i for DM k, $a^k(s_i, s_j) = 1$ indicates that state s_i is absolutely preferred to state s_j for DM k, and $a^k(s_i, s_j) = 0$ indicates that state s_i is equal to the state s_j for DM k.

Definition 12 (*[17]*)**.** If the DM k wish to move from the state s_j to state s_i if and only if $a^k = (s_i, s_j) \geq d_k$. Thus, the fuzzy satisfying threshold of DM k is represent as $d_k(0 < d_k \leq 1)$.

Definition 13 (*[17]*)**.** A given state $s_i \in R_k(s)$ is called a fuzzy unilateral improvement state from state s for DM k if and only if $a^k(s_i, s) \geq d_k$.

Definition 14 (*[17]*)**.** Let $k \in N$ and $s_i, s \in S$. The fuzzy unilateral improvement matrix is an $m \times m$ adjacency matrix, where

$$J_k(s_i, s) = \begin{cases} 1 & (s_i, s) \in A_k \text{ and } a^k(s_i, s) \geq d_k \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The fuzzy unilateral improvement list is represented as $\widetilde{R}_k^+(s) = \{s_i : a^k(s_i, s) \geq d_k\}$.

Remark 3 (*[17]*)**.** When $d_k = 1$ for DM $k \in N$, the definition of a focus DM's fuzzy unilateral improvement list is consistent with the existing definition of a focus DM's (crisp) unilateral improvement list.

3.2. An algorithm for supplementing IFPRs

The condition that IFPR can be supplemented is that it has at least $m - 1$ independent known non-diagonal elements. In other words, each state appears at least once in the known elements of IFPR matrix. A lot of algorithms are proposed for supplementing the IFPR to a complete FPR [34–37]. Among them, Xu, et al. [34] proposed a normalizing rank aggregation method, which is used to calculate preference values for completing the IFPR. Let $w = (w_1, w_2, \ldots, w_m)^T$ be the weighting vector. Then, the preference values "z_{ij}" of IFPR matrix are replaced by $\frac{m}{2}(w_i - w_j) + 0.5$. The elements of auxiliary FPR $\overline{R} = (\overline{r}_{ij})_{m \times m}$ are shown as follow:

$$\overline{r}_{ij} = \begin{cases} r_{ij} & \text{if } r_{ij} \neq z_{ij} \\ \dfrac{m}{2}(w_i - w_j) + 0.5 & \text{otherwise} \end{cases} \tag{5}$$

However, the preference value $\overline{r}_{ij}$ may be out of the range of [0,1]. Let $\overline{r}_{ij} = -g_{ij}$ or $\overline{r}_{ij} = g_{ij} + 1$, where $g_{ij} > 0$. In particular, the maximum value of g_{ij} is denoted as g, denoted as $g = \max g_{ij}$. Faced with this situation, a new matrix $\widetilde{R} = (\widetilde{r}_{ij})_{n \times n}$ is introduced by Herrera-Viedma, et al. [41] based on the matrix $\overline{R} = (\overline{r}_{ij})_{n \times n}$, where

$$\widetilde{r}_{ij} = \frac{\overline{r}_{ij} + g}{1 + 2g}, \quad i, j = 1, 2, \ldots, m \tag{6}$$

Next, we introduce a new algorithm for completing IFPRs.

Based on the above Algorithm 1, it is easy to obtain a complete FPR. Again, the normalizing rank aggregation method is used to derive the weight vector:

$$w_i = \frac{\sum_{j=1}^{m} \overline{r}_{ij}}{m^2/2}, \quad i = 1, 2, \ldots, m \tag{8}$$

4. The model of inverse approach of GMCR with IFPRs

4.1. Basic definitions

In this subsection, some basic definitions for constructing inverse analysis model are introduced.

Definition 15. For DM k, the elements of each row of the reachable matrix J_k are multiplied by the corresponding value of its ordinal score vector, and the result is called forward reachable ordinal score matrix, denoted as J_kV_k.

$$(J_kV_k)(i, j) = \begin{cases} v_j & J_k(i, j) = 1 \text{ and } V_k(s_j) = v_j \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The ith row of the matrix J_kV_k is represented as $e_i^T \cdot (J_kV_k)$. The $(R_kV_k)(s_i)$ represents the forward reachable ordinal score list, denoted as $(R_kV_k)(s_i) = \{v_j : (J_kV_k)(i, j) = v_j \text{ and } v_j \neq 0\}$.

Definition 16. For DM l, the elements of each row of the reachable matrix J_l are multiplied by the corresponding value of its ordinal score vector for opponent DM k, and the result is called inverse reachable ordinal score matrix, denoted as J_lV_k.

$$(J_lV_k)(i, j) = \begin{cases} v_j & J_l(i, j) = 1 \text{ and } V_k(s_j) = v_j \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

The ith row of the matrix J_lV_k is represented as $e_i^T \cdot (J_lV_k)$. The $(R_lV_k)(s_i)$ represents the inverse reachable ordinal score list, denoted as $(R_lV_k)(s_i) = \{v_j : (J_lV_k)(i, j) = v_j \text{ and } v_j \neq 0\}$.

Definition 17. For DM l, the elements of each row of the improvement reachable matrix J_l^+ are multiplied by the corresponding values of its ordinal score vector for opponent DM k, and the result is called inverse improvement reachable ordinal score matrix, denoted as $J_l^+V_k$.

$$(J_l^+V_k)(i, j) = \begin{cases} v_j & J_l^+(i, j) = 1 \text{ and } V_k(s_j) = v_j \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The ith row of the matrix $J_l^+V_k$ is represented as $e_i^T \cdot (J_l^+V_k)$. The $(R_l^+V_k)(s_i)$ represents the inverse improvement reachable ordinal score list, denoted as $(R_l^+V_k)(s_i) = \{v_j : (J_l^+V_k)(i, j) = v_j \text{ and } v_j \neq 0\}$.

4.2. Construction of inverse analysis model

Theorem 1. *Suppose that state s is Nash equilibrium for DM k, and let $V_k(s) = v$, $|R_k(s)| = m_1$, and all the complete ordinal score values of these m_1 improvement reachable states from state s are represented as v_i, $i = 1, \ldots, m_1$, respectively. Therefore, the least constraint conditions that DM k satisfies Nash stability in state s are $v_i < v$, $i = 1, \ldots, m_1$.*

Proof. If state s satisfies Nash equilibrium for DM k. According to Nash stability definition, all the ordinal score values of improvement reachable list from state s are less than the value v. Thus,

$$v_i < v, \quad i = 1, \ldots, m_1 \tag{12}$$

The (12) is the least constraint conditions that DM k satisfies Nash equilibrium in state s.

Theorem 2. *Suppose that state s is GMR equilibrium for DM k, and let $V_k(s) = v$, and $|R_k(s)| = m_1$. If there are p improvement reachable states in m_1 reachable states, they are denoted as s_i, $i = 1, \ldots, p$, and complete ordinal score values of these states are denoted as v_i, $i = 1, \ldots, p$, respectively. If there are q non-improvement reachable states, they are denoted as s_j, $j = 1, \ldots, q$, and complete ordinal score values of these states are denoted as v_j, $j = 1, \ldots, q$, respectively, and $p + q = m_1$. Therefore,*

$$\begin{cases} \min((R_lV_k)(s_1), (R_lV_k)(s_2), \ldots, (R_lV_k)(s_p)) < v \\ v_i > v, & i = 1, \ldots, p \\ v_j < v, & j = 1, \ldots, q \end{cases} \tag{13}$$

Algorithm 1:

Input: IFPR $R = (r_{ij})_{m\times m}$.

Output: Complete FPR $\overline{R} = (\overline{r}_{ij})_{m\times m}$.

Step 1. Applying Eq. (5) to replace the unknown element "z_{ij}" of IFPR using $\frac{m}{2}(w_i - w_j) + 0.5$.

Step 2. Using the normalizing rank aggregation to obtain weights

$$w_i = \frac{\sum_{j=1, j\neq i}^{m} \overline{r}_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} \overline{r}_{ij}} = \frac{\sum_{j=1, j\neq i}^{m} \overline{r}_{ij}}{\frac{m^2}{2}} \tag{7}$$

Next, the Eq. (7) can be expressed as: $Aw = b$, where A is an $m \times m$ matrix and b is a positive m-vector.

Step 3. Solving $Aw = b$, we can obtain the weight vector $w = (w_1, w_2, \ldots, w_m)^T$. Next, we use Eq. (5) to calculate the unknown elements "z_{ij}" of IFPR, and let $\overline{R} = (\overline{r}_{ij})_{m\times m}$.

Step 4. When the elements $\overline{r}_{ij}$ out of the scope [0, 1], we obtain either $\overline{r}_{ij} = -g_{ij}$ or $\overline{r}_{ij} = 1 + g_{ij}$, where $g_{ij} > 0$. Then, we use Eq. (6) to obtain $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$, and let $\overline{R} = \tilde{R}$.

Step 5. Output the complete FPR $\overline{R} = (\overline{r}_{ij})_{m\times m}$.

Proof. According to GMR stability, all the ordinal score values of the improvement reachable states are greater than v, and all the ordinal score values of the non-improvement reachable states are less than v. Thus,

$$\begin{cases} v_i > v, & i = 1, \ldots, p \\ v_j < v, & j = 1, \ldots, q \end{cases} \tag{14}$$

At the same time, for opponent DM l, there exists a reachable list from each improvement reachable state s_i, $i = 1, \ldots, p$, and the complete ordinal score value of at least one state in each reachable list must be less than v. In other words, at least one element in each reachable list $(R_l V_k)(s_i)$, $i = 1, \ldots, p$ is less than v.

$$\min((R_l V_k)(s_1), (R_l V_k)(s_2), \ldots, (R_l V_k)(s_p)) < v \tag{15}$$

Obviously, (14) and (15) need to hold simultaneously to obtain (13), which ensures that the state s satisfies GMR equilibrium. In addition, according to Remark 1, there are 2^{m_1} cases. Thus, we can obtain the least constraint conditions using the same method in each case. Finally, the union of the least constraint conditions in each case, which make the state s be GMR equilibrium for DM k.

In the following paper, note that the symbols p and q have the same characteristic as shown in Theorem 2.

Theorem 3. *Suppose that state s is SEQ equilibrium for DM k, and let $V_k(s) = v$, and $|R_k(s)| = m_1$. Therefore, the least constraint conditions that DM k satisfies SEQ equilibrium in state s are shown as follows.*

$$\begin{cases} \min((R_l^+ V_k)(s_1), (R_l^+ V_k)(s_2), \ldots, (R_l^+ V_k)(s_p)) < v \\ v_i > v, & i = 1, \ldots, p \\ v_j < v, & j = 1, \ldots, q \end{cases} \tag{16}$$

Proof. The definition of SEQ is the same as GMR stability except that while considering the sanction of the focus DM's potential unilateral improvements taken by the opponent, the focus DM just considers the opponent's credible sanctions. Thus, according to SEQ stability, all ordinal score values of the improved reachable states and all ordinal score values of the non-improved reachable states satisfy the following characteristics.

$$\begin{cases} v_i > v, & i = 1, \ldots, p \\ v_j < v, & j = 1, \ldots, q \end{cases} \tag{17}$$

Furthermore, when calculating the inverse analysis model of SEQ, only the credible sanctions of opponent DM are considered. In other words, for opponent DM l, there exists an improvement reachable list from each improvement reachable state s_i, $i = 1, \ldots, p$, and the complete ordinal score value of at least one state in each improvement reachable list must be less than v. In other words, at least one element in each improvement reachable list $(R_l^+ V_k)(s_i)$, $i = 1, \ldots, p$ is less than v.

$$\min((R_l^+ V_k)(s_1), (R_l^+ V_k)(s_2), \ldots, (R_l^+ V_k)(s_p)) < v \tag{18}$$

It is evident that (17) and (18) should hold simultaneously to ensure that the state s satisfies SEQ equilibrium. According to Remark 1, there are 2^{m_1} cases. For each case, we can obtain the least constraint conditions using the same method. Finally, the union of the least constraint conditions in each case, which make the state s be SEQ equilibrium for DM k.

Theorem 4. *Suppose that state s is SMR equilibrium for DM k, and let $V_k(s) = v$, and $|R_k(s)| = m_1$. Thus, the least constraint conditions that DM k satisfies SMR equilibrium in state s are shown in the following.*

$$\begin{cases} \min((R_l V_k)(s_1), (R_l V_k)(s_2), \ldots, (R_l V_k)(s_p)) < v \\ v_i > v, & i = 1, \ldots, p \\ v_j < v, & j = 1, \ldots, q \\ v_h < v, & h = 1, \ldots, u \end{cases} \tag{19}$$

Proof. SMR stability is the same as GMR stability except that the focal DM looks one more step ahead to decide whether to take advantage of a unilateral improvement from the current state. Thus, when calculating the inverse analysis model of SMR, as shown in the proof process of Theorem 2, the first to third inequalities in (19) also can be obtained by using the same analysis solving method. In addition, there exist t reachable states in a set of $R_k(s_i)$, $i = 1, \ldots, p$ and they are denoted as s_x, $x = 1, \ldots, t$. Then, if there are u reachable states in a set of $R_l(s_x)$, $x = 1, \ldots, t$ and they are denoted as s_h, $h = 0, \ldots, u$, and its corresponding complete ordinal score values are v_h, $h = 0, \ldots, u$. All the complete ordinal score values v_h, $h = 1, \ldots, u$ of these reachable states are less than v.

$$v_h < v, \quad h = 1, \ldots, u \tag{20}$$

To ensure that state s satisfies SMR equilibrium, four different inequality conditions should hold simultaneously to obtain (19). Similarly, there are 2^{m_1} cases according to Remark 1. The least constraint conditions in each case can be obtained by using the same method. Finally, the union of the least constraint conditions in each case ensures that the state s is SMR equilibrium for DM k.

In summary, the above four theorems are represented by the mathematical model with the least constraint conditions, which is developed to obtain all the required possible preferences for opponent DM calculating the stability results based on Nash, GMR, SEQ, and SMR definitions. The forward perspective of GMCR includes two stages: modeling and analysis. In contrast, the inverse approach of GMCR explains the condition of how to obtain stability equilibrium results, which involves two stages: modeling and analysis. It is obvious that the difference between standard GMCR (Fig. 1) and inverse approach of GMCR with IFPR (Fig. 2) is the order of steps. In particular, when the preference relation of a focus DM is an IFPR, a new step, called supplementing, must be added. As shown in Fig. 2, the graph shows how to apply inverse approach of GMCR with IFPR. The new version inverse approach of GMCR includes three stages: modeling, supplementing, and analysis. Specifically, the following elements of the conflict need to be entered: DMs, options, infeasible states, allowable transitions, and IFPRs. Finally, based on the basic stability definition, the result of the inverse approach of GMCR is a set of possible preference relations, which are used to ensure a state to be stable.

5. Application of the WEF nexus evaluation conflict in Shandong province

In this section, the WEF nexus evaluation conflict in Shandong province in China is illustrated to show the practicability and correctness of the inverse approach of GMCR with IFPR.

5.1. The background of WEF nexus evaluation in Shandong province

Shandong province is located in the coastal of east China and Jinan city is the capital of Shandong [42]. According to the date of Shandong Statistic Year Book (2017 year), the total gross domestic product (GDP) of Shandong province in 2016 was 6.7 trillion, which was 9.01 percent of national GDP; the population of Shandong province was 9.47 million, accounts for 7.19 percent of the national population. However, the current situation of water resource management in Shandong province is not optimistic. According to the date of Shandong water resources bulletin (2017 year), the total water resource of Shandong in 2016 was 220.32 billion cubic meters, which was 0.67 percent of national water resource, and the per capita water resource was 221.50 cubic meters, which was far lower than the national per capita water resource of 2354.9 cubic meters. According to the index of water shortage, the area with per capita water resources less than 500 cubic meters is considered a severe water scarcity area. It is obvious that the situation of water scarcity in Shandong province is severe. On the other hand, water pollution

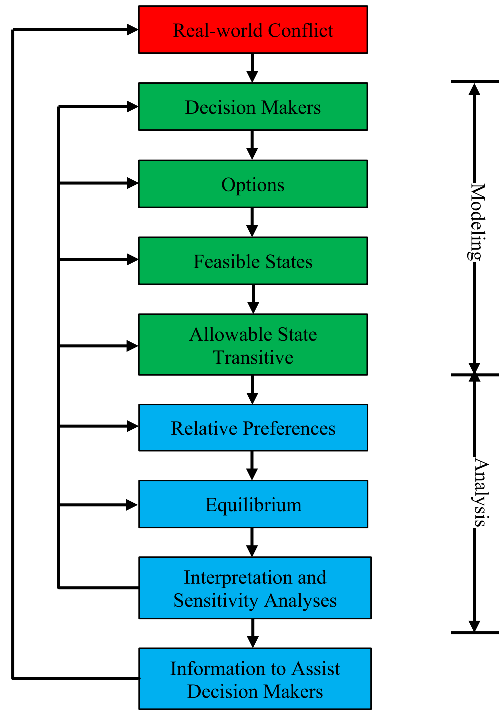

Fig. 1. Procedure of the traditional GMCR methodology.

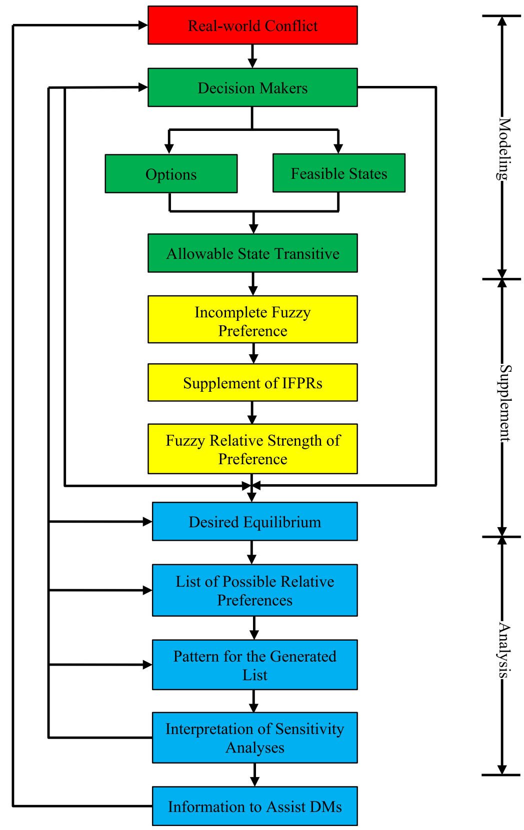

Fig. 2. The general analysis process of inverse GMCR with IFPR.

in Shandong province is also very serious. The standard coal output in Shandong province in 2017 was 13.678 million tons, which accounts for 3.95 percent of the national total. The grain output in Shandong province in 2017 was 47 million tons, which accounts for 7.27 percent of the national total. Energy production and food production have a huge demand for water resources and an important impact on the environment. The production and consumption of WEF nexus are inseparable from the effective economic and social management.

5.2. The descriptions of the WEF nexus evaluation conflict in Shandong province

Nowadays, the rapid growth of the global population and sustained economic development has brought great pressure on resources and the environment, which leads to more research focusing on the complex relationship between water, energy, and food. As described in the above subsection, the internal relationship between water, energy, and food in Shandong province has also become closer. In the management and governance system of coordinating water, energy, and food, the DMs are in different government positions. Apparently, the DMs in certain government departments wish to implement more favorable policies for their interests, which will inevitably lead to conflicts. The WEF nexus evaluation in Shandong province can be seen as a conflict problem.

In the conflict, three DMs, e_1, e_2, and e_3, are work in the Shandong provincial department of water resources, Shandong provincial department of food and energy, and Shandong emergency management office, respectively. Different DMs involve different government departments with different attitudes and actions towards existing resources. The Shandong provincial department of water resources wishes that the energy industry and crop industry can save the use of water resources or charge water resources fees to protect water resources. Many energy companies are worried about a potential economic downturn when stricter water resource policies are implemented. The lack of sufficient water resources in the crop production department will reduce food production. The Shandong provincial department of food and energy is the most noteworthy department which opposes the implementation of strict environmental policies. The energy department wishes to obtain more water resources to facilitate the exploitation of energy. The food crops also need an adequate supply of water resources to ensure food production. At the same time, Shandong emergency management office, which manages and handles emergencies in Shandong province, has tried many methods to resolve the disputes. The emergency management office promotes different departments to obtain a new water use agreement to find a better way to manage and use water resources. In general, the allocation of resources is mostly based on experience and lacks a theoretical basis. Thus, the rational allocation of resources in the WEF nexus evaluation conflict in Shandong province is a key for solving this contradiction.

5.3. The basic components of the graph model

(1) DMs and options: Three DMs from different government departments have different attitudes and policies towards resources. Then, the options of each DM must be identified. The DM e_1 proposes some policies for protecting water resources and reducing water resource consumption. However, the energy and food departments need water to meet their own development objectives. Thus, for the energy and food departments, who use water resources rationally and efficiently would be an effective way to answer the inequality between demand and supply. Especially, when the face of the situation like in irrigation season or energy consumption season, the existing situation does not work well, the DM e_2 will take irrational actions to get more water. Then, the DM e_2 hopes DM e_1 to consider the actual situation and modify relevant policies to increase the water resource to the other DMs and promote each DM development. Thus, for the purpose of water resource protection, the DM e_1 adjusts the existing policies. The DM e_3 has tried to promote a cooperation of different departments to obtain a new water use agreement. Thus, the DMs and options in the WEF nexus evaluation conflict in Shandong province are shown in Table 1.

Table 1
DMs and options of WEF nexus evaluation in Shandong province.

DMs	Options	Explanation
e_1	O_1: Modify	Adjust the existing policies and make cooperate to protect the water resource.
e_2	O_2: Take action	Take irrational actions to get more water
	O_3: Cooperation	Cooperate with other DMs to get more water
e_3	O_4: Promote	Promote different departments to obtain a new water use agreement

(2) Feasible states: As shown in Table 1, there are three DMs and four options in the WEF nexus evaluation conflict in Shandong province. Mathematically, there are $2^4 = 16$ states. We use "Y" to represent that the option is taken by the corresponding DM and "N" is used to represent that the option is not taken by the corresponding DM. Because DM e_3 selects only "Y" on option O_4, some of these 16 states are impossible to achieve. Thus, there are 8 feasible states in Table 2.

Table 2
Feasible states of WEF nexus evaluation conflict in Shandong province.

DMs	Options	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
e_1	O_1	Y	N	N	Y	Y	N	Y	N
e_2	O_2	N	N	Y	N	Y	Y	Y	N
	O_3	N	N	N	Y	N	Y	Y	Y
e_3	O_4	Y	Y	Y	Y	Y	Y	Y	Y

(3) Directed graph: Fig. 3 represents the directed graph of the WEF nexus evaluation conflict in Shandong province. In Fig. 3, each node of the graph represents feasible states, and the different colors of the arcs in the graph represent different DMs. The black arc represents DM e_1, the red arc represents DM e_2. As the options of DM e_3 are fixed and single, the DM e_3 is not displayed in Fig. 3. The direction of arc indicates the achievable state transition. Note that there is only reversible move in the graph.

5.4. The completing process of IFPRs

In the negotiation of WEF nexus evaluation conflict in Shandong province, if the DM e_1 agrees to modify relevant policies, the cooperation can be used not only to develop and utilize water resources but also to promote the development of the region. If DM e_2 agrees to cooperate with DMs e_1 and e_3, it can get the support of the other DMs and obtain more water resources. Particularly, in the energy requirement season, the DM e_2 can exploit more energy and produce more food based on more water resources. In this conflict, the option of DM e_3 is single and definite. DM e_3 wishes to construct the cooperation of different departments to obtain a new water use agreement. Thus, the ordinal score vector of DM e_3 is neglected. However, due to the lack of other professional knowledge and relevant experience, the DM e_1 has an uncertain preference degree of one state over the other state. Therefore, the DM e_1 is considered as the key for solving the WEF nexus evaluation conflict in Shandong province. The IFPR R_1 is introduced to represent the preference relation

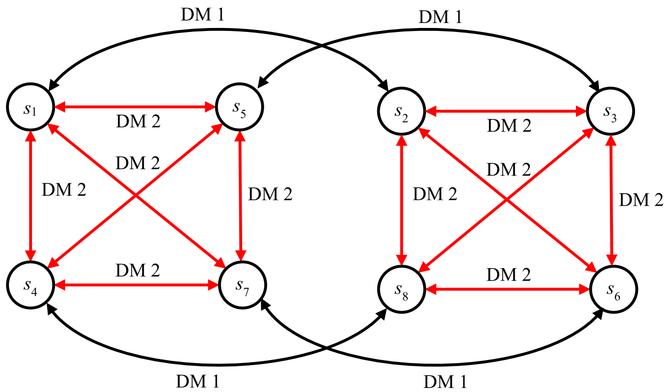

Fig. 3. Directed graph of WEF nexus evaluation conflict in Shandong province.

over all states for DM e_1.

$$R_1 = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \end{array} \\ \left(\begin{array}{cccccccc} 0.5 & 0.3 & 0.3 & 0 & z_{15} & 0.4 & 0.2 & 0 \\ 0.7 & 0.5 & 0.4 & 0.2 & 0.4 & z_{26} & 0.3 & 0.1 \\ 0.7 & 0.6 & 0.5 & 0.3 & 0.6 & 0.4 & z_{37} & 0.1 \\ 1 & 0.8 & 0.7 & 0.5 & 0.8 & 0.8 & 0.6 & z_{48} \\ z_{51} & 0.6 & 0.4 & 0.2 & 0.5 & 0.3 & 0.4 & 0.1 \\ 0.6 & z_{62} & 0.6 & 0.2 & 0.7 & 0.5 & 0.2 & 0.2 \\ 0.8 & 0.7 & z_{73} & 0.4 & 0.6 & 0.2 & 0.5 & 0.3 \\ 1 & 0.9 & 0.9 & z_{84} & 0.9 & 0.8 & 0.7 & 0.5 \end{array}\right) \end{array}$$

Next, we use Algorithm 1 to supplement IFPR and thereby obtain the ordinal score vector from the complete FPR. The vector $w = (w_1, w_2, \ldots, w_8)^T$ is supposed as weight vector of the IFPR for DM e_1.

Step 1. Using $\frac{m}{2}(w_i - w_j) + 0.5$ to replace each unknown element "z_{ij}" of IFPR R_1, and then obtain complete IFPR $\overline{R}_1$.

Step 2. Using the normalizing rank aggregation method, we obtain

$$w_1 = \frac{0.3 + 0.3 + 4(w_1 - w_5) + 0.5 + 0.4 + 0.2}{28} = \frac{1.7 + 4(w_1 - w_5)}{28}$$

$$w_2 = \frac{0.7 + 0.4 + 0.2 + 0.4 + 4(w_2 - w_6) + 0.5 + 0.3 + 0.1}{28} = \frac{2.6 + 4(w_2 - w_6)}{28}$$

$$w_3 = \frac{0.7 + 0.6 + 0.3 + 0.6 + 0.4 + 4(w_3 - w_7) + 0.5 + 0.1}{28} = \frac{3.2 + 4(w_3 - w_7)}{28}$$

$$w_4 = \frac{1 + 0.8 + 0.7 + 0.8 + 0.8 + 0.6 + 4(w_4 - w_8) + 0.5}{28} = \frac{5.2 + 4(w_4 - w_8)}{28}$$

$$w_5 = \frac{4(w_5 - w_1) + 0.5 + 0.6 + 0.4 + 0.2 + 0.3 + 0.4 + 0.1}{28} = \frac{2.5 + 4(w_5 - w_1)}{28}$$

$$w_6 = \frac{0.6 + 4(w_6 - w_2) + 0.5 + 0.6 + 0.2 + 0.7 + 0.2 + 0.2}{28} = \frac{3 + 4(w_6 - w_2)}{28}$$

$$w_7 = \frac{0.8 + 0.7 + 4(w_7 - w_3) + 0.5 + 0.4 + 0.6 + 0.2 + 0.3}{28} = \frac{3.5 + 4(w_7 - w_3)}{28}$$

$$w_8 = \frac{1 + 0.9 + 0.9 + 4(w_8 - w_4) + 0.5 + 0.9 + 0.8 + 0.7}{28} = \frac{5.7 + 4(w_8 - w_4)}{28}$$

It can be rewritten as:

$$\begin{pmatrix} 24 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 24 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 24 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 24 & 0 & 0 & 0 & 4 \\ 4 & 0 & 0 & 0 & 24 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 24 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 24 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{pmatrix} = \begin{pmatrix} 1.7 \\ 2.6 \\ 3.2 \\ 5.2 \\ 2.5 \\ 3 \\ 3.5 \\ 5.7 \end{pmatrix}$$

Step 3. Solving the above equation, we obtain $w_1 = 0.0550$, $w_2 = 0.0900$, $w_3 = 0.1121$, $w_4 = 0.1821$, $w_5 = 0.0950$, $w_6 = 0.1100$, $w_7 = 0.1271$, and $w_8 = 0.2071$. Thus, the weighting vector is:

$$w = (0.0550, 0.0900, 0.1121, 0.1821, 0.0950, 0.1100, 0.1271, 0.2071)^T$$

Then, the missing element of the IFPR $\overline{R}_1$ can be obtained:

$$\begin{aligned} \bar{r}_{15} &= 4 \times (0.0550 - 0.0950) + 0.5 = 0.34, \\ \bar{r}_{51} &= 4 \times (0.0950 - 0.550) + 0.5 = 0.66, \\ \bar{r}_{26} &= 4 \times (0.0900 - 0.1100) + 0.5 = 0.42, \\ \bar{r}_{62} &= 4 \times (0.1100 - 0.0900) + 0.5 = 0.58, \\ \bar{r}_{37} &= 4 \times (0.1121 - 0.1271) + 0.5 = 0.44, \\ \bar{r}_{73} &= 4 \times (0.1271 - 0.1121) + 0.5 = 0.56, \\ \bar{r}_{48} &= 4 \times (0.1821 - 0.2071) + 0.5 = 0.4, \\ \bar{r}_{84} &= 4 \times (0.2071 - 0.1821) + 0.5 = 0.6. \end{aligned}$$

Step 4. The complete FPR $\overline{R}_1$ is:

$$\overline{R}_1 = \widetilde{R}_1 = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \end{array} \\ \begin{pmatrix} 0.5 & 0.3 & 0.3 & 0 & 0.34 & 0.4 & 0.2 & 0 \\ 0.7 & 0.5 & 0.4 & 0.2 & 0.4 & 0.42 & 0.3 & 0.1 \\ 0.7 & 0.6 & 0.5 & 0.3 & 0.6 & 0.4 & 0.44 & 0.1 \\ 1 & 0.8 & 0.7 & 0.5 & 0.8 & 0.8 & 0.6 & 0.4 \\ 0.66 & 0.58 & 0.4 & 0.2 & 0.5 & 0.2 & 0.4 & 0.1 \\ 0.6 & 0.8 & 0.86 & 0.2 & 0.8 & 0.5 & 0.8 & 0.2 \\ 0.8 & 0.7 & 0.56 & 0.4 & 0.6 & 0.2 & 0.5 & 0.3 \\ 1 & 0.9 & 0.9 & 0.6 & 0.9 & 0.8 & 0.7 & 0.5 \end{pmatrix} \end{array}$$

As described above, for ease of calculation, the fuzzy satisfying threshold of the DM e_1 is equal to 1 based on Remark 3. Thus, the complete ordinal score vector can be determined by the weight vector, which can be calculated from the complete FPR using Eq. (8). Then, the weight vector of complete FPR $\overline{R}_1$ for DM e_1 is:

$$w_{e_1} = (0.0635, 0.0944, 0.1138, 0.1750, 0.0950, 0.1488, 0.1269, 0.1969)^T$$

Therefore, the complete ordinal score vector over all states for DM e_1 is equal to $V_1 = (1, 2, 4, 7, 3, 6, 5, 8)$.

5.5. Acquisition of preference relations of opponent DM

The complete ordinal score vector of DM e_1 is determined according to the complete FPR. Then, let $V_2 = (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$ be the ordinal score vector of DM e_2. To achieve a reasonable settlement of the conflict through negotiation, each DM of the conflict not only strives for the maximization of personal interests, but also cannot ignore the interests of the opponent. In the WEF nexus evaluation conflict in Shandong province, the DM e_1 considers that the state s_8 to be the equilibrium state. If the DM e_1 does not consider modifying relevant policies, the DM e_2 will consider taking irritation action to obtain water resources and not consider cooperation. The result leads the state s_8 to state s_5. For DM e_1, it is necessary not only to protect water resources but also to promote cooperation between other DMs. The DM e_1 needs to modify relevant policies, which leads other DMs to obtain benefits and cooperate to jointly develop and utilize water resources. On the other hand, the purpose of the DM e_2 is to obtain more water resources. If DM e_1 can achieve the aim by cooperating with others, the DM e_2 will not take unreasonable actions to obtain water resources. Therefore, the potential equilibrium solution of the conflict should be changed to state s_4. The ultimate goal of DM e_1 is to develop and utilize water resource and the maximum ordinal score value of state s_4 is not equal to 8. Therefore, the ordinal score value of state s_4 for DM e_2 is assumed to be equal to 6, i.e. $V_2(s_4) = 6$.

Inverse analysis of Nash: In Fig. 3, the reachable states of state s_4 for DM e_2 are state s_1, s_5 and s_7. The results show that $m_1 = 3$. According to Theorem 1, the least constraint conditions that the state s_4 satisfies Nash stability are $v_1, v_5, v_7 < 6$. The number of preference rankings satisfying the Nash stability is $C_4^2 A_2^2 A_5^5 = 1440$. It is obvious that the 1440 preference rankings can be calculated at one time.

Inverse analysis of GMR: There are 2^{m_1} cases of preference relations that satisfy the least constraint conditions of GMR stability, i.e. 8 cases. Without loss of generality, we take one of 8 cases as an example. For the state s_4, state s_7 is supposed as an improvement reachable state and state s_1, s_5 are supposed as the non-improvement reachable state. Therefore, according to Theorem 2, we have

$$\begin{cases} v_7 > 6 \\ v_1, v_5 < 6 \\ \min((R_1 V_2)(v_7)) < 6 \end{cases} \tag{21}$$

According to (21), the number of preference rankings satisfying the GMR stability is $C_3^2 A_2^2 A_5^5 = 720$. It is obvious that the 720 preference rankings can be calculated at one time. The remaining 7 cases can be calculated in the same way.

Inverse analysis of SEQ: There are 8 cases of preference relations that satisfy the least constraint conditions of SEQ stability. Without loss of generality, we take one of 8 cases as an example. For the state s_4, state s_7 is supposed as improvement reachable state and state s_1, s_5 are supposed as the non-improvement reachable state. According to Theorem 3, we have

$$\begin{cases} v_7 > 6 \\ v_1, v_5 < 6 \\ \min((R_1^+ V_2)(v_7)) < 6 \end{cases} \tag{22}$$

According to (22), the number of preference rankings satisfying the SEQ stability is $C_4^1 A_5^5 A_2^2 = 960$. At the same time, the remaining 7 cases can be calculated in the same way.

Inverse analysis of SMR: There are 8 cases of preference relations that satisfy the least constraint conditions of SMR stability. Without loss of generality, we take one of 8 cases as an example. For the state s_4, state s_7 is supposed as improvement reachable state and state s_1, s_5 are supposed as the non-improvement reachable state. According to Theorem 4, we have

$$\begin{cases} v_7 > 6 \\ v_1, v_5 < 6 \\ \min((R_1 V_2)(v_7)) < 6 \end{cases} \tag{23}$$

In addition, we consider the ordinal score value of reachable state from the state $R_1(s_7)$ for focus DM. Then, the ordinal vector values of states s_2, s_3, and s_8 satisfy the following inequality:

$$v_2, v_3, v_8 < 6 \tag{24}$$

According to (23) and (24), the number of preference rankings satisfying the SMR stability is $A_5^5 A_2^2 = 240$. At the same time, the remaining 7 cases can be calculated in the same way.

Based on the assumption that state s_4 is the equilibrium state in the conflict problems and the ordinal vector value of state s_4 of DM e_2 is 6, the inverse approach of graph model with IFPR can calculate all the preference relations for DM e_2 ensuring the given state to be equilibrium based on Nash, GMR, SEQ, and SMR stability. In other words, the preference rankings that satisfy four basic stability definitions for DM e_2 are calculated by the proposed inverse analysis method. Thus, if the focus DM has the preference relation of the opponent DM, he can occupy a favorable position in the negotiation, control the development direction of the conflict through some auxiliary methods, and find some creative strategies to reasonably and effectively solve the conflict problems. Moreover, the above proposed method is general and can be used to obtain all the preference rankings when the ordinal score value of state s_4 is not equal to 6 for DM e_2. By analyzing the above results, for example, the preference ranking of DM e_2 according to GMR stability is described as follows: the ordinal score value of state s_7 is great than state s_4, the ordinal score value of state s_1, s_5 are less than state s_4. Thus, if DM e_1 and e_3 master the preference relation that DM e_2 satisfies the GMR stability in state s_4. Then, DM e_1 and e_3 can find some creative strategies to promote DM e_2 to reach the state s_4, which can more reasonably and effectively solve the WEF nexus evaluation conflict in Shandong province. Specifically, when DM e_1 agrees to modify water policies and DM e_3 wishes to promote cooperation to obtain a new water use agreement. In such a case, DM e_2 will choose to cooperate with other DMs to obtain more water resources instead of taking illegal actions to obtain water resources, which will promote the stable development of resources and agricultural departments in Shandong province.

5.6. Implications of the research

In the process of solving the WEF nexus evaluation conflict in Shandong province, we realize that the key to solving the conflict problem is to minimize the loss of resources and protect the existing natural environment while supporting rapid economic development. However, it is difficult to achieve environmental protection and rational allocation of resources only by promulgating environmental policies and regulations. Thus, the goal can be realized through the effective management of DMs in different governmental departments. The construction and improvement of the management capacity of DMs in conflict problems can ensure the effective implementation and operation of environmental management and policy system. The above described research result shows that the inverse approach of graph model with incomplete fuzzy preferences can provide effective information to improve the management ability of DMs, which can better deal with WEF nexus evaluation conflict problems.

In the WEF nexus evaluation conflict in Shandong province, the preference relation of the Shandong provincial department of food and energy is very important for the Shandong provincial department of water resources to change the current situation and improve its management. Particularly, the preference relations over states for the Shandong provincial department of food and energy are calculated by using proposed inverse method. The analysis results can help the Shandong provincial department of water resources changes the existing strategy and improve the management ability, including working with the other DMs to develop and utilize water resources. The Shandong provincial department of food and energy cannot take illegal actions but should cooperate with other DMs to get more water resources. Thus, the WEF nexus evaluation in Shandong province conflict can be perfectly solved, and each DM can achieve his own goals.

5.7. Limitation of the research

There are some deficiencies in the research work of this paper. Specifically, the required preference relation for each DM satisfying stability definitions is not accurate, which is only determined by the limited constraints. In addition, in the WEF nexus evaluation conflict, most DMs work in different government departments. These DMs are supposed as "rational players", thereby ignoring the influence of power asymmetry between different DMs. Of course, the knowledge gap promotes the further development of this research work.

6. Comparison of computational complexity

Sakakibara, et al. [19] firstly emphasized the importance of the inverse problem of graph model. Kinsara, et al. [20] proposed the enumeration method to calculate the preference information of each DM. Furthermore, as described in [22], three different preference relations have been determined to describe the DM's preference attitude. In this paper, we propose a method to obtain preference information for opponent DM in conflict based on the IFPR. Then, the maximum value of computational complexity of the enumeration method for given DM is equal to $(m-1)!$, where m is the number of feasible states in conflict problems. The maximum value of computational complexity determined by the matrix representation method is equal to 1 or $3^{(m-1)}$. The maximum value of computational complexity calculated by our proposed method is 1 or 2^{m_1}. Table 3 shows the difference in the computational complexity and completeness of different methods when calculating the preference for a given DM.

As shown in Table 3, m_1 represents the number of states in the reachable list, which is determined from the given states s for focus DM k by using one-step movement. Compared with the value m of the maximum number of all states, m_1 is definitely less than m. On the one hand, the maximum value of computational complexity of the proposed method is much less than the value of computation complexity of other existing inverse analysis methods. On the other hand, compared with the existing inverse approach of the graph model, the proposed method ensures the completeness of the preference information, that is, all the required preference relations can be obtained at one time.

7. Conclusions and future work of research

In this paper, the WEF nexus evaluation problem is considered as a special type of conflict problem, which is analyzed and resolved by the inverse approach of the GMCR with IFPR. On the one hand, each DM in the WEF nexus evaluation conflict problems adjusts strategy according to inverse analysis results to improve management ability and achieve the goals. On the other hand, as illustrated by the WEF nexus evaluation conflict in the Shandong province, the new inverse approach of the graph model is a more flexible and practical method, which can deal with conflict problems with uncertainty preferences. Especially, the complete FPR can be obtained from the IFPR for DM by using the proposed algorithm. The calculation expression of the inverse analysis approach with IFPR is represented by the mathematical inequality system, which can be formulated to determine the possible preference relations required by opponent DM to realize basic stability definitions.

In summary, some features of the proposed method are shown in the following. (1) The proposed method integrates the IFPR into the inverse approach of the graph model. Thus, it combines the advantages of IFPR and the graph model and then can model and analyze the uncertainty in real-life conflict problems. (2) In general, the research of WEF nexus mainly concentrates on engineering and system. However, in this paper, the WEF nexus is studied from the perspective of management by using the inverse approach of the graph model, which provides a new way to research similar problems in other research scopes. (3) Compared with the existing inverse approach of the graph model, this paper studies and extends the inverse approach of the graph model with incomplete preferences by using the mathematical inequality model, which can enrich the research scope of the GMCR from a different perspective.

In the future work, we will introduce some points that are still unsolved in the research of this paper and deserve further study: (1) According to the mathematical inequality model calculated in this paper, a detailed relevant computational algorithm should be constructed to calculate the preference relations, which satisfy the basic stability definitions. (2) In order to further narrow the scope of the solution and obtain a more accurate preference relation. We need to study other constraints and then add them to the mathematical model obtained in Section 4. (3) Based on the existing research of this paper, when the DMs of conflict problems have power asymmetry, how to obtain preference relation to achieve a stable state and find creative strategies to coordinate the interests of all DMs have important research meaning.

Table 3
The difference in computational complexity of different methods.

Methods	Sakakibara, et al. [19]	Kinsara, et al. [20]	Wang, et al. [22]	The proposed method
Computational complexity	Logical analysis	$(m\text{-}1)!$	1 or $3^{(m-1)}$	1 or 2^{m_1}
Completeness	No	Yes	Yes	Yes

CRediT authorship contribution statement

Dayong Wang: Writing – original draft. **Jing Huang:** Software, Methodology. **Yejun Xu:** Revision, Supervision. **Nannan Wu:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Nomenclature

WEF	Water–Energy–Food
GMCR	Graph Model for Conflict Resolution
DMs	Decision Makers
FPR	Fuzzy Preference Relation
IFPR	Incomplete Fuzzy Preference Relation
IFPRs	Incomplete Fuzzy Preference Relations
Nash	Nash Stability
GMR	General Metarationality
SMR	Symmetric Metarationality
SEQ	Sequential Stability
GDP	Gross Domestic Product

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